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Models for robust tactical planning in multi-stage production systems with uncertain demands

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ABSTRACT

We consider the problem of designing robust tactical production plans, in a multi-stage production system, when the periodic demands of the finished products are uncertain. First, we discuss the concept of robustness in tactical production planning and how we intend to approach it. We then present and discuss three models to generate robust tactical plans when the finished-product demands are stochastic with known distributions. In particular, we discuss plans produced, respectively, by a two-stage stochastic planning model, by a robust stochastic optimization planning model, and by an equivalent deterministic planning model which integrates the variability of the finished-product demands. The third model uses finished-product average demands as minimal requirements to satisfy, and seeks to offset the effect of demand variability through the use of planned capacity cushion levels at each stage of the production system. An experimental study is carried out to compare the performances of the plans produced by the three models to determine how each one achieves robustness. The main result is that the proposed robust deterministic model produces plans that achieve better trade-offs between minimum average cost and minimum cost variability. Moreover, the required computational time and space are by far less important in the proposed robust deterministic model compared to the two others.

1. Introduction

Tactical production planning is a medium-term planning process of production capacity and employment levels to balance supply and demand. The term tactical or "aggregate" implies that the planning is carried out for product types (i.e., product families) rather than for individual finished products [15]. This step is crucial in the planning of production since it bridges the transition from the strategic planning level to the operational planning level. When devising a tactical plan, only projected demands, projected capacities, and projected costs of the various planning options are commonly available to the planner. Conversely, among the control variables available to the planner are adjustments in output rate, employment level, overtime or under-time, and sub-contracting. The main goal of tactical planning is of course to achieve output objectives at the lowest possible total cost including the extra cost resulting from the necessary re-planning. In addition to translate the tactical plan into meaningful production elements, it must be disaggregated (i.e., broken down into specific product requirements) to determine actual labor, material, and inventory requirements. This last stage is very critical since, at this point, the tactical plan may turn out to be infeasible due to an inadequate disaggregation or to unexpected variability in some critical planning parameters (such as demand, lead times, equipment, etc.). It is thus essential that the produced optimal tactical plan be guaranteed robust to the variability of some selected critical planning parameters and feasible at the operational level.

The increasing market pressure on the manufacturers combined with the natural variability of the critical planning parameters makes the issue of "robustness" become an essential aspect in production planning. Therefore, when developing a model for tactical production planning, in particular for multi-stage production systems, two important aspects should be given necessary attention. The first consists in insuring that the produced tactical plan can be disaggregated into at least one detailed feasible plan for the realized demand; and the second consists in insuring that this detailed plan is also feasible at the operational level when the production is converted into jobs to be scheduled. The first aspect was tackled in [10,13,20]. The authors have produced sufficient and necessary conditions for a tactical plan to disaggregate into at least one detailed plan that is feasible for some demand realizations. The second issue was also approached by many authors such as [7,27]. The authors presented models which take into account explicitly some scheduling constraints at the

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tactical level to insure feasibility of the detailed plan at the operational level. In this paper, we propose an equivalent deterministic model for tactical production planning when finished-product demands are stochastic. This model attempts to produce tactical plans which can be disaggregated into at least one feasible detailed plan that is also feasible at the scheduling level. Specifically, the model integrates the variability of the finished-product demands through the use of appropriate planned capacity cushion levels at the various stages of the production system; and it integrates the temporal restrictions through an explicit incorporation of constraints that monitor the production movements from a stage to the following.

The remainder of the paper is organized as follows: in Section 2, we discuss the concept of robustness and how it is approached in the existing literature. In Section 3, we present the deterministic model for the tactical production planning problem and discuss its particular features. In Section 4, we present and discuss the two-stage stochastic and then the robust stochastic optimization based planning models. In Section 5, we discuss the equivalent deterministic model for robust tactical planning. In Section 6, we introduce a design of experiment and conduct a comparative analysis of these three models.

2. Uncertainty and robustness in production planning

Uncertainty may be defined as the difference between the amount of information required to perform a task and the amount of information already possessed [12]. Along the years many researches attempted to formalize and model uncertainty in manufacturing systems [28,31]. Production planning related literature is very rich of approaches and models that were proposed to cope with different forms of uncertainty. At the tactical planning level, production plans generated with models integrating effect of uncertainty are called robust plans. Roughly defined, a robust production plan is the one that remains valid (i.e., feasible or inexpensive to turn into feasible) regardless of the variability resulting from the uncertainty inherent to the production process. In particular, the variability that results from supply and production lead times, from finished-product demands, from prices and production costs, and from deterioration or unexpected failures of the production equipment. Tactical production planning models which do not integrate the variability of these critical parameters in the planning process result often in worthless plans or at the best in plans which must be revised frequently. These repeated alterations usually deteriorate dramatically the initial performance of the generated tactical plan.

Early initiatives to model uncertainty in the production planning context started with some attempts to model demand randomness in inventory management systems using mainly stochastic processes; then in production planning and in particular in hierarchical planning, using stochastic programming, see for example [5,8,9,11,17]. The issue of disaggregation in hierarchical planning when demands are uncertain was also studied by authors such as [3,6,13,20]. Lasserre and Merce [20] introduced the concepts of robustness and consistency in hierarchical planning when aggregate demands are known with certainty but finished-product demands may vary within known intervals. In this context a robust tactical plan is defined as a plan that results in at least one feasible disaggregation for any demand realizations of the finished products. Gfrerer and Zapfel [13] generalized the results of Lasserre and Merce, but they still make the same particular assumption that demands of the product families are known with certainty (deterministic) but demands of the finished products may assume values between known demand lower and upper bounds. Thompson and Davis [29] and Thompson et al. [30] presented an integrated approach to model the uncertainties present in aggregate production planning. They formulated a linear programming model in which the uncertainty in costs, capacities, lead times and demand were modeled using Monte Carlo simulation techniques. They evaluated six production strategies but no disaggregation process was carried out.

Mulvey et al. [26] proposed another approach, called robust optimization, to achieve robust solutions for problems with uncertain parameters. Their approach aims at designing solutions that exhibit low variability of the key performance measure (mainly the cost). The main objective is to produce solutions having a smallest possible expected performance deviation from the performances of the scenario-specific optimal solutions. Mulvey et al. [26] modified the Markowitz [23] two-stage stochastic model by adding a measure of variability of the objective function of the second-stage to the objective function of the first-stage. Depending on the value of the weight put on this variability, the optimization process may favor solutions with higher total expected performance and smaller second-stage performance variance to solutions with lower total expected performance and possibly larger second-stage performance variance. This approach was used for power systems capacity planning [22], for chemical-process planning under uncertainty [2], for telecommunications-network design [4,19], and for many other problems [18]. Ahmed et al. [1] treated the stochastic lot-sizing problem as a sub-problem of the capacity expansion problem. Leung et al. [21] formulated the aggregate production planning under uncertainty and made use Mulvey's framework to solve the problem. They proposed a two-stage stochastic integer programming to solve the problem. Recently, a branch and cut algorithm to solve stochastic lot-sizing problems under demand uncertainty is proposed [14]. The problem is formulated into a multi-stage stochastic integer programming. For an extensive review of production planning models under uncertainty, we refer the readers to [25].

In the remainder of the paper we present a two-stage stochastic programming model, a robust stochastic optimization based model, and an equivalent deterministic model for tactical production planning. These models focus mainly on modeling the uncertainty inherent to the finished-product demands. Of course they can be extended to include other uncertain parameters such as lead times, equipment reliability, prices and costs. These extensions can be straightforward or more complex depending on the modeling approach that is used. For example, if the model is scenario based then additional parameters increase only the number of scenarios to consider, but the model remains fundamentally the same.

3. The tactical production planning problem

The tactical planning problem is concerned with determining optimal amounts of internal regular and overtime capacities to be assigned to each product-family, at each stage of the manufacturing system, so that the individual demands of the finished products are satisfied over the planning horizon H. Inventories are to be kept minimal, and the cost function to be optimized consists in total fixed costs (setup cost for internal production) incurred for each production lot, a linear internal production and external sub-contracting costs, and linear holding costs assessed, respectively, on the inventory levels at the end of each period. The challenge in this planning process consists in determining, optimally, the periodic production capacity levels assigned to each product family at each production stage, based on aggregated forecasts of finished-product demands, while insuring that there will always exist a feasible disaggregation of this available production capacity level to satisfy each demand realizations of the finished products, and that there will always exist a feasible schedule of this disaggregation at the operational level. The following sub-section presents a deterministic model of the aggregate planning problem when all parameters are known with certainty. Note that in our model we do not consider the sub-contracting possibility. It can of course be allowed without additional difficulty.

To formally define the aggregate planning model being discussed, we let P be the set of product families to be produced during a W-periods planning horizon H. Each period w (typically a week) is sub-divided into T sub-periods t (typically days). We assume that each product family $k \in P$ consists of a set of finished products denoted by N_k. Each individual product $j \in N_k$ has a periodic (weekly) demand denoted by $d_{jk}(w)$ in each period w of the planning horizon H. The periodic (weekly) demands for each product family k, expressed in required capacity time units at each stage of the production system, may then be obtained as $\sum_{j\in N_k} a^l_{jk} d_{jk}(w)$, where a^l_{jk} is the amount of capacity in time units required to produce one unit of the finished product $j \in N_k$ at stage $l = 1, \ldots, L$, where L is the last stage.

3.1. The tactical production planning model when detailed demands are known with certainty

If we assume that the periodic demands $d_{jk}(w)$ of each finished product $j \in N_k$ are known with certainty, the tactical production planning problem can be formulated as a deterministic mixed integer linear program. To define the remaining model parameters, we let $c_{jk}(w)$ be the variable cost of producing the finished product $j \in N_k$ during the period w; and $h_{jk}(w)$ be the holding cost for one unit of finished product $j \in N_k$ carried from period w to period $w+1$. We also let $f^l_k(t,w)$ be the fixed cost of setting up the system at stage l for the processing of the finished products of the family k during the sub-period t of the period w; and $\theta^l(t,w)$ be the per time unit variable cost of overtime capacity at stage l in the sub-period t of the period w. Finally, we let $\kappa^l_R(t,w)$ and $\kappa^l_O(t,w)$ be, respectively, the planned regular and overtime capacities at stage l in sub-period t of the period w, expressed in time units.

To define the model variables, we let $Q^l_k(t,w)$ be the amount of capacity, in time units, reserved for production of the finished products of the family k at stage l in sub-period t of the period w. We let $O^l(t,w)$ be the additional overtime production capacity, also in time units, made available at stage l in sub-period t of the period w for all product families. We let $y^l_k(t,w)$ be a binary variable assuming 1 if the production of a finished product of the family k takes place during the sub-period t of the period w. We also let $x^l_{jk}(t,w)$ be the quantity of the finished product $j \in N_k$ processed in stage l during the sub-period t of the period w. The quantity $\sum_{t=1}^{T} x^L_{jk}(t,w)$ is thus the amount of finished product $j \in N_k$ ready to be shipped in period w. Finally, we define the variable $i_{jk}(w)$ to be the inventory level of finished product $j \in N_k$ carried from period w to period $w+1$. The following mixed-integer linear program provides a model for the general deterministic tactical planning problem denoted by TPP_D:

$$\text{Minimize } Z^D_{TP} = \sum_{w=1}^{W} (Z_{prod}(w) + Z_{cap}(w))$$

Subject to:

$$\sum_{k\in P} Q^l_k(t,w) - O^l(t,w) \leqslant \kappa^l_R(t,w), \quad \forall t,w,l \tag{1}$$

$$O^l(t,w) \leqslant \kappa^l_O(t,w), \quad \forall t,w,l \tag{2}$$

$$Q^l_k(t,w) - (\kappa^l_R(t,w) + \kappa^l_O(t,w)) y^l_k(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{3}$$

$$\sum_{j\in N_k} a^l_{jk} x^l_{jk}(t,w) - Q^l_k(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{4}$$

$$\sum_{\nu=1}^{w-1} \sum_{\tau=1}^{T} (x^{l-1}_{jk}(\tau,\nu) - x^l_{jk}(\tau,\nu)) + \sum_{\tau=1}^{t-1} x^{l-1}_{jk}(\tau,w) - \sum_{\tau=1}^{t} x^l_{jk}(\tau,w) \geqslant 0, \quad \forall j,k,t,w,l \tag{5}$$

$$\sum_{t=1}^{T} x^L_{jk}(t,w) + i_{jk}(w-1) - i_{jk}(w) = d_{jk}(w), \quad \forall j,k,w \tag{6}$$

$$Q^l_k(t,w), O^l(t,w), x^l_{jk}(t,w), i_{jk}(w) \geqslant 0, \quad y^l_k(t,w) \in \{0,1\} \quad \forall j,k,t,w,l$$

where

$$Z_{prod}(w) = \sum_{k\in P} \sum_{j\in N_k} \left(c_{jk}(w) \sum_{t=1}^{T} x^L_{jk}(t,w) + h_{jk}(w) i_{jk}(w) \right)$$

and

$$Z_{cap}(w) = \sum_{t=1}^{T} \sum_{l=1}^{L} \left(\sum_{k\in P} f^l_k(t,w) y^l_k(t,w) + \theta^l(t,w) O^l(t,w) \right)$$

Constraints (1) and (2) are capacity restrictions; they make sure that the amounts of regular and overtime capacities in each stage that are assigned during each sub-period do not exceed the available capacity limits. Constraints (3) guarantee that the appropriate setup costs, in each stage during each sub-period, are paid whenever a finished product of a given family is processed. Constraints (4) assure that the total quantity of the finished products of a given family, processed in a given stage during a given sub-period, does not exceed the capacity reserved for that family. Constraints (5) are consistency restrictions, they guarantee that the quantity of a finished product that is processed, during a given sub-period in a given stage, has already been processed in the previous stage during the previous periods. We assume here that the quantity processed during a given sub-period in a given stage moves to the next stage only at the end of that sub-period. It can therefore be processed in the next stage only in the next sub-period. Finally, constraints (6) are the usual flow conservation constraints that balance supply and demand; they define the quantities of each finished product to be produced during each period.

3.2. Model discussion and hypothesis

The model discussed in this paper generalizes the tactical planning model that we developed for a manufacturer specializing in medical (X-ray) and graphical film production. The actual production system has a multi-stage structure operating 7 days a week 16 h a day in two shifts. Some days and shifts are considered to be overtimes. Demand forecasts are typically prepared on a weekly basis. In this case, the manufacturer has precise figures for the fixed costs of mobilizing the capacity at each stage of the system for the production of a given product family. There are of course minor setup costs (setup times) related to the finished products of each family. These last costs were generally difficult to obtain, but their corresponding setup times could be measured. These setup times were then left to the operational level phase which deals with daily production scheduling during each sub-period.

This tactical production planning model is, however, general enough and is convenient for many manufacturing systems. Unlike the usual tactical planning models available in the literature, this proposed model has two distinguishing characteristics. The first is related to the fixed cost that is linked, in this case, to the amount of capacity reserved to a given product family in a given production stage of the system. Finished-product setup costs are assumed to be unimportant and are neglected. The second characteristic is related to the temporal aspect in a multi-stage production system. We choose to model this aspect explicitly at the tactical level through the use of constraints (5). These constraints monitor the movements of the production from a stage to the next to assure feasibility at the operational level. This guarantees that the quantities which are planned to be shipped in a period can be effectively scheduled and completed by the end of that period. Now, there is one technical problem related to the initial stocks at the beginning of the planning horizon that must be addressed. We suggest as a possible solution to let the initial stocks be equal to the demands of the first period(s) and to force the model to produce these demands at the end of the planning horizon. This way the issue of initial stocks is resolved without falsifying the actual performances of the produced plans.

4. Stochastic and robust optimization models for the tactical production planning problem

The difficulty in producing reliable forecasts for finished-product demands forces the production planners to consider robust production plans. Usually, exact demands are discovered only after the production plan has been put into execution. It is therefore crucial to develop models that are capable of generating robust plans requiring only few occasional adjustments. In the following paragraph we present two models taking into account the stochastic aspects of the demand. The first model is the usual two-stage stochastic programming model and the second is based on the robust optimization paradigm.

4.1. A two-stage stochastic optimization model for tactical planning

To reformulate the above deterministic tactical planning model (TPP_D) in a stochastic setting, we assume that demands $d_{jk}(w)$ are stochastic parameters with known distributions. We use bold face for the random variables in order to distinguish them from their particular realizations ($\mathbf{d}$ represents the random variable while d stands for its particular realization). The objective of the stochastic planning model is to minimize the current total cost of capacity reservation (family fixed and planned overtime costs) and expected future production and holding costs. The resulting model (TPP_S) is given as follows:

$$\text{Minimize } Z_{TP}^S = \sum_{w=1}^{W} Z_{cap}(w) + E(Z(Q,\mathbf{d}))$$

Subject to:

$$\sum_{k \in P} Q_k^l(t,w) - O^l(t,w) \leqslant \kappa_R^l(t,w), \quad \forall t,w,l \tag{7}$$

$$O^l(t,w) \leqslant \kappa_O^l(t,w), \quad \forall t,w,l \tag{8}$$

$$Q_k^l(t,w) - (\kappa_R^l(t,w) + \kappa_O^l(t,w))y_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{9}$$

$$Q_k^l(t,w), O^l(t,w) \geqslant 0, \quad y_k^l(t,w) \in \{0,1\}, \ \forall k,t,w,l$$

where $Z_{cap}(w) = \sum_{t=1}^{T}\sum_{l=1}^{L}\left(\sum_{k\in P} f_k^l(t,w)y_k^l(t,w) + \theta^l(t,w)O^l(t,w)\right)$, and the cost function $Z(Q,\mathbf{d})$ is a random variable obtained as the optimal value of the following problem:

$$Z(Q,\mathbf{d}) = \text{Minimize } \sum_{w=1}^{W}\sum_{k\in P}\sum_{j\in N_k}\left(c_{jk}(w)\sum_{t=1}^{T} x_{jk}^L(t,w) + h_{jk}(w)i_{jk}(w)\right) + \sum_{w=1}^{W}\sum_{k\in P}\sum_{j\in N_k}(p_{jk}(w)r_{jk}(w))$$

Subject to:

$$\sum_{j\in N_k} a_{jk}^l x_{jk}^l(t,w) - Q_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{10}$$

$$\sum_{\nu=1}^{w-1}\sum_{\tau=1}^{T}(x_{jk}^{l-1}(\tau,\nu) - x_{jk}^l(\tau,\nu)) + \sum_{\tau=1}^{t-1} x_{jk}^{l-1}(\tau,w) - \sum_{\tau=1}^{t} x_{jk}^l(\tau,w) \geqslant 0 \quad \forall j,k,t,w,l \tag{11}$$

$$\sum_{t=1}^{T} x_{jk}^L(t,w) + i_{jk}(w-1) - i_{jk}(w) + r_{jk}(w) = d_{jk}(w), \quad \forall j,k,w \tag{12}$$

$$x_{jk}^l(t,w), i_{jk}(w), r_{jk}(w) \geqslant 0, \quad \forall j,k,t,w,l$$

To insure that the flow conservation constraints are always satisfied in case of stochastic demands, we added new variables $r_{jk}(w)$ returning the amounts of unsatisfied demands of the finished product $j \in N_k$ in a given period w. These variables appear in the objective function with costs $p_{jk}(w)$ which can be interpreted as a penalty of not satisfying a demanded unit of the finished product $j \in N_k$ in a given period w. This penalty can be chosen to be at least equal to the market price of the finished product $j \in N_k$ in period w. In addition, the optimal value $Z(Q,\mathbf{d})$ of the second-stage problem (10)–(12) is a random variable which can be obtained as a function of the first-stage decision variables $Q_k^l(t,w)$ and the random demand $\mathbf{d}$. The expectation in the objective function (Z_{TP}^S) is taken with respect to the probability distribution of $\mathbf{d}$.

At this point, there are two important remarks that must be highlighted; the first is related to the complexity of the two parts of the model. Observe that the master part of the stochastic model is a mixed integer linear programming problem. This problem has a reasonable number of variables and is not impacted by the number of scenarios that are considered in the solution process. The sub-problems part which must be solved to obtain $E(Z(Q,\mathbf{d}))$ depend on the number of scenarios that are considered. This number may be very large, fortunately the sub-problems are linear programs and can be solved efficiently.

4.2. A robust stochastic optimization model for tactical planning

Mulvey et al. [26] modified the two-stage stochastic programming model by adding a weighted measure of variability of the second-stage objective function to the objective function of the first stage. Varying the weight put on this variability forces the optimization process to produce solutions that may present higher expected total costs with lower second-stage cost-deviations. This paradigm is referred to as robust optimization.

To describe the robust stochastic optimization based model for tactical planning, we let $\Omega = \{1,2,\ldots,S\}$ be the finite set of possible demand scenarios, each with a positive probability of occurrence p_s, and we let $d_{jk}^s(w)$ denote the demand, under scenario $s \in \Omega$, of a finished product $j \in N_k$ in period w. We also let $x_{jk}^{s,l}(t,w)$ be the quantity of finished product $j \in N_k$ produced in the sub-period t of the period w, $i_{jk}^s(w)$ the inventory level of finished product $j \in N_k$ carried from period w to period $w+1$, and $r_{jk}^s(w)$ the proportion of the demand of the finished product $j \in N_k$ that is not satisfied in period w under scenario $s \in \Omega$. The following mixed-integer linear program provides a model for the stochastic robust tactical planning model and is denoted by (TPP_R) with the objective of minimizing the function Z_{TP}^R:

$$\text{Minimize } Z_{TP}^R = \eta \cdot \max_{s\in\Omega}(\xi_s - \xi_s^*) + \lambda \cdot \sum_{s\in\Omega} p_s \xi_s$$

Subject to:

$$\sum_{k\in P} Q_k^l(t,w) - O^l(t,w) \leqslant \kappa_R^l(t,w), \quad \forall t,w,l \tag{13}$$

$$O^l(t,w) \leqslant \kappa_O^l(t,w), \quad \forall t,w,l \tag{14}$$

$$Q_k^l(t,w) - (\kappa_R^l(t,w) + \kappa_O^l(t,w)) y_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{15}$$

$$\sum_{j\in N_k} a_{jk}^l x_{jk}^{s,l}(t,w) - Q_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l,s \tag{16}$$

$$\sum_{v=1}^{w-1}\sum_{\tau=1}^{T}(x_{jk}^{s,l-1}(\tau,v) - x_{jk}^{s,l}(\tau,v)) + \sum_{\tau=1}^{t-1} x_{jk}^{s,l-1}(\tau,w) - \sum_{\tau=1}^{t} x_{jk}^{s,l}(\tau,w) \geqslant 0, \quad \forall j,k,t,w,l,s \tag{17}$$

$$\sum_{t=1}^{T} x_{jk}^{s,L}(t,w) + i_{jk}^s(w-1) - i_{jk}^s(w) + r_{jk}^s(w) = d_{jk}^s(w), \quad \forall j,k,w,s \tag{18}$$

$$Q_k^l(t,w), O^l(t,w), x_{jk}^{s,l}(t,w), i_{jk}^s(w), r_{jk}^s(w) \geqslant 0, \quad y_k^l(t,w) \in \{0,1\} \quad \forall j,k,t,w,l,s$$

The cost function ξ_s^* is the optimal value obtained by solving the deterministic model TPP_D with periodic demands $d_{jk}^s(w)$ realized under scenario $s \in \Omega$. The cost function ξ_s is the optimal cost resulting from the occurrence of scenario $s \in \Omega$ given the robust values of variables $Q_k^l(t,w)$ and $O^l(t,w)$:

$$\xi_s = \sum_{w=1}^{W} (Z_{prod}^s(w) + Z_{cap}(w))$$

where

$$Z_{prod}^s(w) = \sum_{k\in P}\sum_{j\in N_k}\left(c_{jk}(w)\sum_{t=1}^{T} x_{jk}^{s,L}(t,w) + h_{jk}(w) i_{jk}^s(w) + p_{jk}(w) r_{jk}^s(w)\right)$$

and

$$Z_{cap}(w) = \sum_{t=1}^{T}\sum_{l=1}^{L}\left(\sum_{k\in P} f_k^l(t,w) y_k^l(t,w) + \theta^l(t,w) O^l(t,w)\right)$$

The weights, η and λ, are two parameters which can be freely set by the planner. These parameters actually reflect the planner's preferences. If one wishes to produce plans with low variability but higher expected cost, he has to increase the weight of η and vice versa.

Both models TPP_S and TPP_R provide thus solutions that have higher chances of being valid for a large number of scenarios. The solutions provided by the model TPP_S tend to minimize the average total cost while those provided by the model TPP_R are of min–max type solutions. The concern with both models is that their solutions are very expensive to obtain in terms of computational time and space.

5. A robust deterministic model for the tactical production planning problem

The model we are proposing in this section extends the idea of safety stock used in inventory management to production planning. Since we are dealing with planning at the tactical level, what is actually being "stocked" is an amount of production capacity sufficient to cope with the variability of the demand. The robust deterministic model we are proposing consents in the plans that it generates some extra capacity which is made available to cover almost all realized demands with a certain level of confidence. Naturally, it is expected that the proposed model provides optimal plans which perform almost as good as those obtained with other models but if possible with smaller performance variability. The following paragraphs discuss the model in detail.

Assume that each periodic demand $d_{jk}(w)$ is stochastic with a known demand distribution $F(\overline{d}_{jk}(w), \sigma_{jk}(w))$, where $\overline{d}_{jk}(w)$ and $\sigma_{jk}(w)$ are its average and standard deviation, respectively. Assume also that these periodic demands are independent. Thus, for any set of consecutive periods $H_{vw} = \{v, v+1, \dots, w\}$, the cumulative demand $d_{jk}^{vw} = \sum_{u=v}^{w} d_{jk}(u)$ has a probability distribution with average and standard deviation given, respectively, by

$$\overline{d}_{jk}^{vw} = \sum_{u=v}^{w} \overline{d}_{jk}(u) \quad \text{and} \quad \sigma_{jk}^{vw} = \left(\sum_{u=v}^{w} (\sigma_{jk}(u))^2\right)^{1/2}$$

Now, suppose that a service level of $100(1-\alpha)\%$ is to be achieved. If the plan call for a quantity that is to be available in the beginning of period v to cover the realized demands in the consecutive periods v, $v+1$, to w, with $v \leqslant w$, then based on the standard deviation of the cumulative demand d_{jk}^{vw} a capacity cushion level that needs to be foreseen to achieve this service level is given by $\delta_{jk}^{vw} = \theta_\alpha \sigma_{jk}^{vw}$, where θ_α reflects the required service level, i.e., $P(d_{jk}^{vw} \leqslant \overline{d}_{jk}^{vw} + \theta_\alpha \sigma_{jk}^{vw}) = 1-\alpha$.

To describe the robust deterministic model for tactical production planning we introduce new binary variables z_{jk}^{vw}, defined for each pair of periods (v,w) with $v \leqslant w$. The variable z_{jk}^{vw} assumes value 1 if a production of finished-product $j \in N_k$ takes place in period v to cover integrally the realized demands in all periods u from v to w, and 0 otherwise. We denote the realized demands from v to w by $D_{jk}^{vw} = (\overline{d}_{jk}^{vw} + \theta_\alpha \sigma_{jk}^{vw})$. The model, denote by TPP_{RD}, can then be formulated as follows:

$$\text{Minimize } Z_{TP}^{RD} = \sum_{w=1}^{W} (Z_{prod}(w) + Z_{cap}(w))$$

Subject to:

$$\sum_{k\in P} Q_k^l(t,w) - O^l(t,w) \leqslant \kappa_R^l(t,w), \quad \forall t,w,l \tag{19}$$

$$O^l(t,w) \leqslant \kappa_O^l(t,w), \quad \forall t,w,l \tag{20}$$

$$Q_k^l(t,w) - (\kappa_R^l(t,w) + \kappa_O^l(t,w)) y_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{21}$$

$$\sum_{j\in N_k} a_{jk}^l x_{jk}^l(t,w) - Q_k^l(t,w) \leqslant 0, \quad \forall k,t,w,l \tag{22}$$

$$\sum_{v=1}^{w-1}\sum_{\tau=1}^{T}(x_{jk}^{l-1}(\tau,v) - x_{jk}^{l}(\tau,v)) + \sum_{\tau=1}^{t-1} x_{jk}^{l-1}(\tau,w) - \sum_{\tau=1}^{t} x_{jk}^{l}(\tau,w) \geqslant 0 \quad \forall j,k,t,w,l \tag{23}$$

$$\sum_{t=1}^{T} x_{jk}^{L}(t,w) + i_{jk}(w-1) - i_{jk}(w) = \overline{d}_{jk}(w), \quad \forall j,k,w \tag{24}$$

$$\sum_{t=1}^{T} x_{jk}^{L}(t,v) + i_{jk}(v-1) - D_{jk}^{vw} z_{jk}^{vw} \geqslant 0, \quad \forall j,k,v,w; v \leqslant w \tag{25}$$

$$\sum_{u=1}^{w}\sum_{v=w}^{W} z_{jk}^{uv} = 1, \quad \forall j,k,w \tag{26}$$

$$\sum_{t=1}^{T} a_{jk}^L x_{jk}^L(t,v) - \left(\sum_{t=1}^{T}(\kappa_R^L(t,v) + \kappa_O^L(t,v))\right)\sum_{w=v}^{W} z_{jk}^{vw} \leqslant 0, \quad \forall j,k,v \tag{27}$$

$$Q_k^l(t,w), O^l(t,w), x_{jk}^l(t,w), i_{jk}(w) \geqslant 0, z_{jk}^{vw}, \quad y_k^l(t,w) \in \{0,1\}, \ \forall j,k,t,w,l$$

Constraints (25) make certain that if production takes place in period v to cover demands of the periods $u \in H_{vw} = \{v, v+1, \dots, w\}$, then the amount to be produced augmented by the inventory in the beginning of period v must be sufficient to achieve the required service level (i.e., to cover $\overline{d}_{jk}^{vw} + \theta_\alpha \sigma_{jk}^{vw}$). Constraints (26) make sure that the variables z_{jk}^{vw} decompose the planning horizon into a partition of subsets, of consecutive periods, in which the production takes place only in the first period. Constraints (27) guarantee that variables z_{jk}^{vw}

assume a value of 1, for some $w \in H$ with $v \leqslant w$, only if the quantity $\sum_{t=1}^{T} a_{jk}^{L} x_{jk}^{L}(t,v)$ is positive for $v \in H$.

One of the main benefits of the model is that it takes advantage of the accuracy resulting from cumulative demand forecasts and uses some readily available planning parameters such as demand averages and standard deviations to produce plans that are capable of achieving the required service levels. This model generates tactical plans for which we can guarantee, with some level of confidence, disaggregation and scheduling feasibility. In the following paragraph we undertake a comparative analysis to evaluate how good are the plans obtained with the above robust deterministic model in contrast with those produced with the stochastic and robust optimization models.

6. The design of experiment and computational result

To assess the quality of the solutions obtained by each of the three models, we compared the performances of the plans produced by the robust deterministic model (TPP_{RD}) to those produced by the two-stage stochastic optimization model (TPP_S) and the robust stochastic optimization model (TPP_R). We developed a design of experiments that includes some critical planning parameters thought likely to have a significant impact on the produced solutions. Since we are typically interested in evaluating how the expected cost and variability of the plans generated by the robust deterministic model (TPP_{RD}) compare to the expected cost and variability of the plans generated by the other two models (TPP_S) and (TPP_R), we focused on the impact of demand variability and capacity tightness on the solutions produced. The experiments are carried out in two phases. In the first phase, we have kept the size of the planning horizon and the number of stages fixed, and we limited the experiment to a single family with a single finished product to obtain a reasonable number of scenarios only for the purpose of comparing the models. In a second phase, we carried some additional tests with multi-family and multi-product problems. Solving the proposed new robust deterministic model (TPP_{RD}) was straightforward. However, solving the (TPP_S) and (TPP_R) models was challenging due to the very large number of scenarios. Actually, just reading the scenario requires quite some time. This could be tackled by taking samples of scenarios and conducting the experiments. This is not done in this paper, we wanted to show that the scenario based models (TPP_S) and (TPP_R) may not be useful for practical problems. We did not provide any computational times when the computer could not find a solution.

6.1. The design of experiment

As mentioned above, in this experiment we investigate the impact of demand variability and capacity tightness on the solutions produced by the three models (TPP_S), (TPP_R), and (TPP_{RD}). We consider three levels of demand variability corresponding, respectively, to low, medium, and high demand variability. For each demand variability level, we consider three levels of capacity tightness corresponding to situations with loose, moderately loose and tight capacity.

In the design of experiment, we consider a two-stage production system a 6-period horizon planning. The cost structure is generated as follows: the setup cost $f^l(t,w)$ is selected randomly from interval [750,1000], the internal cost $c(w)$ is selected randomly from [5,10], and the market price is obtained as $p_w = 2(c_w + \sum_{l=1}^{L} \min_t(f^l(t,w))/\min_w(\bar{d}_{jk}(w)))$. The value of the holding cost is given by $h(w) = \delta c(w)$ where δ is uniformly distributed between 0.15 and 0.25. The period-by-period average demands $\bar{d}(w)$ are selected randomly from [150, 250]. Further, for each demand $d(w)$ a standard deviation $\sigma(w)$ is selected randomly from three different ranges [1,5], [1,25] and [1,50] corresponding, respectively, to low, medium and high variability of finished-product demands.

The capacity is the last parameter selected in each test problem. We first introduce the target average utilization of capacity factor β to define the capacity tightness. The factor β is set to 0.55, 0.75 and 0.95 corresponding, respectively, to situations with loose, moderately loose and tight capacity constraints. We first compute the part-period ratios $(\sum_t \min_l (f^l(t,w))/h(w))$ for each period, and then we take the average of these values. The average part-period ratio is then divided by the average of the periodic demands. This provides us with the number of periods that the part-period ratio covers on average to obtain a good approximation of the incapacitated optimal solution. To strengthen the effect of the capacity we tighten this number by multiplying it by 0.75. The tightened number is then multiplied by the average demand. The internal capacity in each period $\kappa(w)$ is then obtained by dividing the last result by the target average utilization of capacity factor β. We assume that 70% of this internal capacity is for regular capacity and 30% is for over time capacity, where the cost per unit over time capacity is 1.5 times the cost for regular capacity. The cost for regular capacity is defined as $\min_t (f^l(t,w))/\kappa(w)$.

6.2. Computational result analysis

For each demand variability level, and capacity tightness level we generated five test problems. We then constructed, for each test problem, a set of highly probable demand scenarios defined as follows: first, for each demand $d(w)$ three possible values are selected $\{\bar{d}(w) - 2\sigma(w), \bar{d}(w), \bar{d}(w) + 2\sigma(w)\}$ for which the probabilities are given, respectively, by $P(\bar{d}(w) - 3\sigma_t \leqslant d(w) \leqslant \bar{d}(w) - 1\sigma(w))$, $P(\bar{d}(w) - 1\sigma(w) \leqslant d(w) \leqslant \bar{d}(w) + 1\sigma(w))$, and $P(\bar{d}(w) + 1\sigma(w) \leqslant d(w) \leqslant \bar{d}(w) + 3\sigma(w))$. We assume a normal distribution for each periodic demand which gives us the probability for the three possible demand values as $\{0.16, 0.68, 0.16\}$, respectively. Since the planning horizon contains six periods, we obtain a total of 729 ($=3^6$) scenarios for each test problem.

We solve the three models (TPP_S), (TPP_R), and (TPP_{RD}) for each test problem data, using ILOG OPL programming with Cplex 10.1, and determine their respective optimal solutions. Let $(Q,O)_S$, $(Q,O)_R$, and $(Q,O)_{RD}$ be, respectively, the scenario independent components of these optimal solutions. These variables are the ones which are usually decided in advance and cannot be changed by the planner, the other remaining variables, namely the actual quantities of each finished product, are the recourse variables which may be modified depending on demand realizations.

Furthermore, for each test problem (i.e., for the same periodic demand averages and standard deviations) we generated randomly $ns=100$ demand realization selected from the range $[\bar{d}(w) - 3\sigma(w), \bar{d}(w) + 3\sigma(w)]$. We then resolve the problem for each demand realization keeping the components $(Q,O)_S$, $(Q,O)_R$, and $(Q,O)_{RD}$ from the three models (TPP_S), (TPP_R), and (TPP_{RD}) fixed or considered as known parameters. We resolve all the problems using deterministic model (TPP_D) from Section 3 allowing modification for unsatisfied demands, i.e., replacing constraint (6) with

$$\sum_{t=1}^{T} x_{jk}^{L}(t,w) + i_{jk}(w-1) - i_{jk}(w) + r_{jk}(w) = d_{jk}(w), \quad \forall j,k,w \tag{28}$$

The scheme for the experimental design and performance evaluation is summarized in Fig. 1.

This provides us with the cost resulting from each new scenario realization given the reserved capacities at each stage for each product family. We denote by $Z^S(s)$, $Z^R(s)$, and $Z^{RD}(s)$, respectively, the actual values obtained by each model for each scenario $s \in NS = \{1,\dots,ns\}$. We then compute for each model

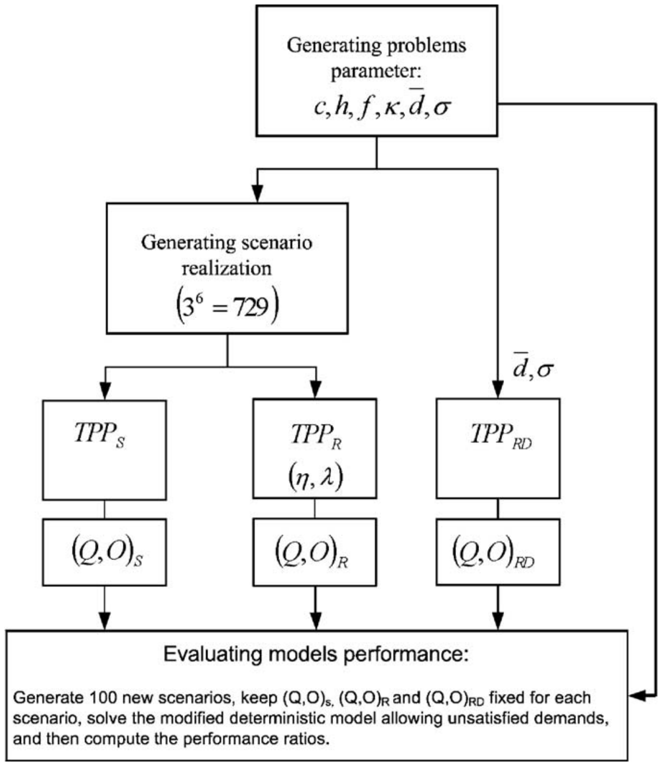

Fig. 1. The models evaluation scheme.

the resulting average value and variability denoted, respectively, by $(\bar{Z}^S, \nu^S)$, $(\bar{Z}^R, \nu^R)$, and $(\bar{Z}^{RD}, \nu^{RD})$. This variability is obtained by $\sqrt{\nu^S = (1/(ns-1))\sum_{s=1}^{ns}(Z^S(s)-\bar{Z}^S)^2}$, $\sqrt{\nu^R = (1/(ns-1))\sum_{s=1}^{ns}(Z^R(s)-\bar{Z}^R)^2}$, and $\sqrt{\nu^{RD} = (1/(ns-1))\sum_{s=1}^{ns}(Z^{RD}(s)-\bar{Z}^{RD})^2}$.

Now, to compare the performance of the three models we compute the following average performance gap ratios:

$$APG^S = 100\frac{\bar{Z}^{RD} - \bar{Z}^S}{\bar{Z}^{RD}}\% \quad \text{and} \quad APG^R = 100\frac{\bar{Z}^{RD}_{\alpha} - \bar{Z}^R}{\bar{Z}^{RD}}\%$$

with the following performance variability ratios:

$$PVR^S = \frac{\nu^S}{\nu^{RD}} \quad \text{and} \quad PVR^R = \frac{\nu^R}{\nu^{RD}}$$

The performance characteristics selected for the analysis are thus the gap ratios between the average cost of the solutions obtained with the robust deterministic model and the average costs of the solutions obtained, respectively, with the two-stage stochastic and the robust stochastic optimization models. As shown in the following tables, when these gaps are small, the deterministic robust model has found solutions that are as good as those obtained with the two-stage stochastic and the robust stochastic optimization models. This happens generally for problems with loose to moderately loose capacity constraints and low to medium demand variability. When the variability of the demand increases the gap also increases slightly reflecting the fact that the solutions provided by the deterministic robust optimization model are a bit expensive than the solutions obtained by the two-stage stochastic and the robust stochastic optimization models. This result is intuitively expected since the deterministic robust optimization model will often provide solutions with higher costs, but more stable than the solutions obtained by the two-stage stochastic and the robust stochastic models. One of the essential results that must be mentioned is that the gap ratios are not higher than 6% (see Tables 1, 2 and 3, high demand variability) when comparing the model (TPP_{RD}) with the model (TPP_S) and 4% (see Tables 1, 2 and 3, high demand variability) with (TPP_R). The effect of demand variability on the gap ratios is quite significant especially when capacity is loose. When demand variability is low, the model (TPP_{RD}) performs almost as good as the model TPP_S (i.e., gap 0.3%) (see Tables 1 and 2, low demand variability) and performs even better than the model (TPP_R) (i.e., gap −2.6%) (see Table 1, low demand variability). However, when demand variability is high, the gap ratios deteriorates down to almost 5% for both TPP_S and TPP_R (see Table 1, high demand variability).

We also measured the ratios of the variability of the costs of the solutions resulting, respectively, from the two-stage stochastic and the robust stochastic optimization models to the variability of the cost of the solutions obtained with the robust deterministic model. In general, the model (TPP_R) performs better in term of variability. There is no clear pattern whether capacity tightness and demand variability play important roles in this phenomenon. It is expected, however, that the min–max procedure keeps the solution within a certain range which gives us a smaller variability. When the capacity is tight (see Table 3), (TPP_{RD}) performs as good as (TPP_R) in terms of variability (i.e. ratio 0.97 in case of low demand variability and ratio 0.92 in case of high demand variability). It is worthwhile to note that the model (TPP_{RD}) performs better than TPP_S when the variability of demand is high in tight and moderately loose capacity. The standard deviation of the solution from the model TPP_S is almost 1.1 (see Table 3, high demand variability) times the standard deviation from the (TPP_{RD}) or approximately 10% higher. In loose capacity, however, the model TPP_S outperforms the model (TPP_{RD}).

To conclude, based on this relatively limited design of experiment, the proposed robust deterministic model (TPP_{RD}) seems to provide quite good solutions. It generates solutions with the average costs that are only 6% worse than that of the solutions obtained with the stochastic and 4% with the robust stochastic model. The model (TPP_{RD}) performs well in term of variability when demand variability is high and when the capacity is tight or moderately tight. When capacity is loose, although the variability of the solution worsen, the average value is better than the model (TPP_R) and a small gap, i.e., 0.31% with the model (TPP_S). We may say that the performance of the model (TPP_{RD}) is a kind of trade-off between the (TPP_S) which emphasizes average cost values and (TPP_R) which emphasizes variability of the solution. Moreover, the required computational time and space are by far less important in the robust deterministic model (TPP_{RD}) compared to the two models (TPP_S) and (TPP_R).

6.3. Computational time

As mentioned above, solving the proposed deterministic robust model was straightforward even for practical size problems. The stochastic and robust stochastic optimization models, however, require enormous time and space. For even modest size problems, the computational time required for these scenario-based models could not be determined. If P is the set of product families and N_k is the set of finished products for each family $k \in P$, then the number of finished products can be denoted as $N = card(\bigcup_{k\in P} N_k)$, where $\bigcup_{k\in P} N_k$ is the set of all finished products. If we let W be the length of the horizon planning and assume that for each finished product we consider only three scenario realizations for each period, then we will have a full combination tree consisting of $3^{(W.N)}$ scenarios. Table 4 shows computational times for each combination of N and W when these times are available. All models were solved using ILOG Cplex 10.1 on a 3 GHz CPU and 1 GB RAM processor. We encountered some computational problems when solving the scenario-based models, e.g. when the number of items is 2 and the length of the planning horizon is 6 periods, the number of scenario grows exponentially to more than a half million scenarios. This consumes an enormous amount of memory space.

Table 1
Computational results in cases of loose capacity.

Dev.	TPP_S		TPP_R		TPP_{RD}		Relative $\frac{Z_S}{Z_{RD}}$		Relative $\frac{Z_R}{Z_{RD}}$	
	$\bar{Z}_S$	v^S	$\bar{Z}_R$	v^R	$\bar{Z}_{RD}$	v^{RD}	$\frac{\bar{Z}_S}{\bar{Z}_{RD}}$	$\frac{v^S}{v^{RD}}$	$\frac{\bar{Z}_R}{\bar{Z}_{RD}}$	$\frac{v^R}{v^{RD}}$
Low	13 072	78	13 088	74	13 097	87	0.19	0.90	0.07	0.85
	13 482	46	13 497	47	13 495	55	0.10	0.83	−0.01	0.85
	12 925	68	12 933	64	12 943	74	0.14	0.93	0.07	0.88
	13 103	62	14 075	58	13 151	98	0.36	0.63	−7.03	0.59
	12 609	63	13 474	53	12 703	96	0.74	0.66	−6.07	0.66
Average							0.31	0.79	−2.60	0.77
Medium	13 786	303	13 861	279	14 539	563	5.18	0.54	4.67	0.50
	11 617	709	11 982	291	12 026	366	3.41	1.94	0.37	0.79
	13 520	302	13 558	281	14 688	516	7.95	0.58	7.69	0.55
	12 005	170	12 910	167	12 081	215	0.63	0.79	−6.86	0.78
	10 272	224	10 352	224	10 302	236	0.29	0.95	−0.49	0.95
Average							3.49	0.96	1.08	0.71
High	13 559	980	13 642	914	14 115	823	3.94	1.19	3.35	1.11
	12 273	545	12 328	501	12 624	582	2.78	0.94	2.35	0.86
	10 945	460	10 989	411	12 036	687	9.06	0.67	8.70	0.60
	13 478	905	13 621	699	13 395	948	−0.62	0.95	−1.69	0.74
	12 719	411	12 801	406	13 955	453	8.85	0.91	8.27	0.89
Average							4.80	0.93	4.20	0.84

Table 2
Computational results in cases of moderately loose capacity.

Dev.	TPP_S		TPP_R		TPP_{RD}		Relative $\frac{Z_S}{Z_{RD}}$		Relative $\frac{Z_R}{Z_{RD}}$	
	$\bar{Z}_S$	v^S	$\bar{Z}_R$	v^R	$\bar{Z}_{RD}$	v^{RD}	$\frac{\bar{Z}_S}{\bar{Z}_{RD}}$	$\frac{v^S}{v^{RD}}$	$\frac{\bar{Z}_R}{\bar{Z}_{RD}}$	$\frac{v^R}{v^{RD}}$
Low	15 244	70	15 292	86	15 320	123	0.50	0.57	0.18	0.70
	15 648	63	15 662	56	15 736	119	0.56	0.53	0.47	0.47
	11 253	73	11 268	64	11 265	91	0.11	0.81	−0.02	0.71
	15 907	59	15 929	56	15 953	85	0.29	0.69	0.15	0.65
	14 382	54	14 385	53	14 393	70	0.08	0.77	0.05	0.75
Average							0.31	0.67	0.17	0.65
Medium	12 780	827	12 856	198	12 991	376	1.63	2.20	1.04	0.53
	14 464	305	14 517	287	14 760	357	2.01	0.85	1.65	0.80
	13 976	417	14 675	301	14 863	600	5.97	0.70	1.26	0.50
	16 163	196	16 172	184	16 413	241	1.52	0.81	1.47	0.76
	13 059	480	13 161	248	13 179	358	0.91	1.34	0.14	0.69
Average							2.41	1.18	1.11	0.66
High	13 270	843	14 004	658	14 072	807	5.70	1.04	0.48	0.82
	16 834	1047	16 883	958	16 229	907	−3.73	1.16	−4.03	1.06
	11 330	934	11 439	682	11 531	761	1.75	1.23	0.80	0.90
	13 550	985	14 260	725	14 930	857	9.24	1.15	4.49	0.85
	12 457	491	12 775	400	12 567	598	0.87	0.82	−1.65	0.67
Average							2.77	1.08	0.02	0.86

The fact that the computational time and space for the scenario-based models are impractical to our real problem makes the proposed robust deterministic model a much more meaningful alternative. As a deterministic model, the robust deterministic model (TPP_{RD}) provides a good solution within a reasonable computational time. In our practical problem, we have five product families, each family has 20 finished products manufactured in a two-stage production line ($N = 100$ finished products). The length of the planning horizon, W, is 6 weeks. The robust deterministic model (TPP_{RD}) needed 5 min to find an optimal solution, which is acceptable, especially because the company needs to review its planning on a weekly basis. For the same amount of time, it is impossible to solve the problem using the scenario-based models unless if the number of scenario is reduced significantly. There are some papers in the literature discussing ways to reduce the number of scenario using sample generation (for an extensive survey see [16]). However, the accuracy gained from reducing the number of scenario deteriorates significantly. Kaut and Wallace [16] show that a scenario-based model requires some minimum number of scenarios, which could be very large, to work effectively. Furthermore, they argue that there is no single scenario generator which fits all models even if these models were subject to the same phenomena.

Table 3
Computational results in cases of tight capacity.

Dev.	TPP_S		TPP_R		TPP_{RD}		Relative $\frac{Z_S}{Z_{RD}}$		Relative $\frac{Z_R}{Z_{RD}}$	
	$\bar{Z}_S$	ν^S	$\bar{Z}_R$	ν^R	$\bar{Z}_{RD}$	ν^{RD}	$\frac{\bar{Z}_S}{\bar{Z}_{RD}}$	$\frac{\nu^S}{\nu^{RD}}$	$\frac{\bar{Z}_R}{\bar{Z}_{RD}}$	$\frac{\nu^R}{\nu^{RD}}$
Low	12 315	53	13 225	53	12 325	69	0.08	0.77	−7.30	0.77
	14 618	84	14 639	64	14 647	92	0.20	0.91	0.05	0.69
	14 789	117	15 015	197	15 312	140	3.42	0.84	1.94	1.41
	19 449	224	19 449	224	19 936	218	2.44	1.03	2.44	1.03
	14 054	106	14 074	97	14 055	103	0.00	1.03	−0.14	0.94
Average							1.23	0.92	−0.60	0.97
Medium	16 535	1028	17 005	763	17 576	1085	5.92	0.95	3.25	0.70
	14 135	1506	14 613	540	14 691	612	3.78	2.46	0.53	0.88
	14 298	836	14 380	715	15 772	855	9.35	0.98	8.83	0.84
	14 153	660	14 523	628	14 365	601	1.48	1.10	−1.10	1.05
	14 739	425	14 793	420	15 288	567	3.59	0.75	3.24	0.74
Average							4.82	1.25	2.95	0.84
High	12 636	848	13 988	490	14 170	824	10.82	1.03	1.28	0.59
	14 531	938	14 661	1067	15 128	896	3.95	1.05	3.09	1.19
	11 738	689	12 308	547	12 763	569	8.03	1.21	3.56	0.96
	15 319	716	15 411	765	15 471	889	0.98	0.81	0.39	0.86
	11 406	818	11 796	567	11 821	574	3.50	1.42	0.21	0.99
Average							5.46	1.10	1.70	0.92

Table 4
Computational time.

N	$W=4$			$W=6$		
	TPP_S	TPP_R	TPP_{RD}	TPP_S	TPP_R	TPP_{RD}
1	5″	1′	0.25″	34″	3′	0.25″
2	1 h 37′	na[a]	0.25″	na	na	50″
100	na	na	5′	na	na	5′

[a]Not available.

7. Conclusions

In this paper we presented three alternative models to generate production plans that are robust to the variability resulting from demand uncertainty. The first is a two-stage stochastic optimization model, the second is a robust stochastic optimization model, and the third is a robust deterministic model. The results show that, the solutions obtained with the proposed robust deterministic model are of quite good quality. The average performances of the solutions, obtained with this robust deterministic model, are only 5% more expensive than those obtained with the two-stage stochastic and the robust stochastic optimization models. However, the performance variability of the solutions produced by the deterministic robust model is very small compared to that of the solution produced by the two-stage stochastic or the robust stochastic optimization models. Thus, when the stability of the solution's performance is the primary criteria of the planner, our suggested model is a good alternative planning model to use instead of the robust stochastic model.

Also, observe that for practical reasons usually a finite set of scenarios, typically those supposed to occur with high probabilities, is used as a basic set of possible scenarios for the two-stage stochastic and robust stochastic optimization models. This implies that the solutions obtained by these two models are robust with respect to these selected scenarios. Of course, this does not mean that other scenarios will not occur. Actually, when other scenarios occur we may end up with plans that are very expensive to adapt or review. Here again, the proposed robust deterministic model resolves partly this concern, and it can be extended to include other types of variability such as that resulting from uncertainty of lead times and costs without major modifications.

One last important issue discussed in this paper but was not tested against the proposed model is the issue of disaggregation. Two approaches were proposed, respectively, by Lasserre and Merce [20] and Gferer and Zapfel [13] to assure existence of at least one feasible disaggregation. The approach proposed by Lasserre and Merce [20] assumes no fixed costs at the aggregate level and adds constraints to the model to assure a required minimum inventory level in the beginning of each period. In many practical cases there are very important fixed costs even at the aggregate level. For these cases the approach shows its limitations. The approach proposed by Gfrerer and Zapfel [13] is in general difficult to put in practice. It requires dynamic demand scheme which are not easy to obtain. One of the perspectives is to determine how this issue can be tackled with the proposed robust deterministic model. The few initial tests with modified constraints proposed by Lasserre and Merce [20] seem to offer some good results.

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