Simulation of nonlinear surface waves generated by submarine landslides

S. S. Tjandra and S. R. Pudjaprasetya

Citation: AIP Conference Proceedings **1716**, 020008 (2016); doi: 10.1063/1.4942991
View online: http://dx.doi.org/10.1063/1.4942991
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1716?ver=pdfcov
Published by the AIP Publishing

Simulation of Nonlinear Surface Waves Generated by Submarine Landslides

S.S. Tjandra[1,2,a)] and S.R. Pudjaprasetya[1]

[1]*Industrial & Financial Mathematics Research Group, Bandung Institute of Technology, Bandung, Indonesia*
[2]*Industrial Engineering, Parahyangan Catholic University, Bandung, Indonesia*

[a)]Corresponding author: ssudharmatjandra@yahoo.com

Abstract. In this work, we study about nonlinear surface wave generated by submarine landslides. The landslide is modeled as a rigid block sliding down along a sloping bottom. We implement a numerical scheme based on the staggered finite volume method to simulate surface wave. We enhance the conservative scheme for nonlinear shallow water equation (SWE) to include this bottom motion. Our numerical result show a good agreement with previous results of the nonlinear boundary integral equation model (BIEM) by Lynett and Liu and central-upwind scheme by Kurganov and Petrova.

INTRODUCTION

Water waves generated by landslides are natural phenomena that occur under some condition such as earthquake, erosion, and heavy rainfalls. In this work, we study submarine landslide. The sea surface deforms in response to the movement of the landslide on a sea floor. The landslide movement can generate several types of powerful and destructive long waves. In some conditions, submarine landslide can cause tsunamis.

The study of landslide motion generating surface wave has long been an interesting subject of researches, see for instance Heinrich [1] and Watts [2, 3]. Here, we utilize the numerical scheme that approximate the nonlinear shallow water equation. This method was first introduced by Stelling and Duinmeijer in [6]. In this paper, the scheme is used to study free surface wave generated by an under water mass sliding down on a constant slope. Once the wave is formed, then it propagates through the sea and kept going towards the shore. In this contribution, we simulate water wave generated by two landslides object. First, underwater landslide is geometrically idealized by a fully submerged semi-ellipse like a hump on the sloping bottom. This is a typical landslide motion commonly used in literatures [2, 4, 5] as a benchmark test of numerical schemes. Second, we also simulate surface wave generated by triangular landslide. This typical landslide is studied in [1, 7].

STAGGERED CONSERVATIVE SCHEME

Consider the nonlinear shallow water equation, reads as

$$h_t + (hU)_x = 0, \tag{1}$$

$$U_t + UU_x + g\eta_x = 0. \tag{2}$$

where $U(x,t)$ denotes the depth averaged horizontal velocity of fluid particles, $h(x,t) = \eta(x,t) + d(x,t)$ denotes the water thickness, $\eta(x,t)$ is the surface elevation and $d(x,t)$ is the bottom topography.

In this section, we give a resume of the staggered scheme that we use for solving the nonlinear SWE. Full description of this method can be found in [6]. Consider a computational domain $[0, L]$ with a staggered partition points $x_{\frac{1}{2}} = 0, x_1, \cdots, x_{i-\frac{1}{2}}, x_i, x_{i+\frac{1}{2}}, \cdots, x_{Nx+\frac{1}{2}} = L$. Continuity equation (1) is approximated at cell $[x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$ and

Application of Mathematics in Industry and Life
AIP Conf. Proc. 1716, 020008-1–020008-6; doi: 10.1063/1.4942991

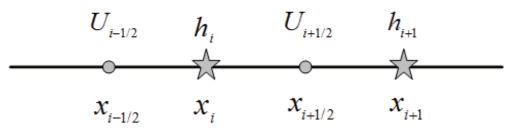

FIGURE 1. Staggered grid with configuration of calculated variables h dan U.

momentum equation (2) is approximated at cell $[x_i, x_{i+1}]$. The approximate equations are

$$\frac{dh_i^n}{dt} + \frac{{}^*h_{i+\frac{1}{2}}^n U_{i+\frac{1}{2}}^n - {}^*h_{i-\frac{1}{2}}^n U_{i-\frac{1}{2}}^n}{\Delta x} = 0, \tag{3}$$

$$\frac{dU_{i+\frac{1}{2}}^n}{dt} + g\frac{\eta_{i+1}^{n+1} - \eta_i^{n+1}}{\Delta x} + (UU_x)_{i+\frac{1}{2}}^n = 0. \tag{4}$$

In this staggered formulation η and h are computed at every full grid points x_i, whereas u is computed at every half grid points $x_{i+\frac{1}{2}}$, see Figure 1. Approximation for h values at staggered points is the following upwind approximation

$$^*h_{i+\frac{1}{2}} = \begin{cases} h_i\,, & \text{if } U_{i+\frac{1}{2}} \geq 0 \\ h_{i+1}\,, & \text{if } U_{i+\frac{1}{2}} < 0. \end{cases} \tag{5}$$

For the advection term $(UU_x)_{i+\frac{1}{2}}$ we implement the momentum conservative approximation as introduced by G.S. Stelling and Duijnmeijer in [6], which is as follows. Writing the advection term as

$$UU_x = \frac{1}{h}\left(\frac{\partial(qU)}{\partial x} - U\frac{\partial q}{\partial x}\right), \tag{6}$$

with $q = hU$ denotes the horizontal momentum, a consistent approximation of the advection (6) is

$$(UU_x)_{i+\frac{1}{2}} = \frac{1}{\bar{h}_{i+\frac{1}{2}}}\left(\frac{\bar{q}_{i+1}\,{}^*U_{i+1} - \bar{q}_i\,{}^*U_i}{\Delta x} - U_{i+\frac{1}{2}}\frac{\bar{q}_{i+1} - \bar{q}_i}{\Delta x}\right), \tag{7}$$

with

$$\bar{h}_{i+\frac{1}{2}} = \frac{1}{2}(h_i + h_{i+1}), \quad \bar{q}_i = \frac{1}{2}(q_{i+\frac{1}{2}} + q_{i-\frac{1}{2}}), \quad q_{i+\frac{1}{2}} = {}^*h_{i+\frac{1}{2}}U_{i+\frac{1}{2}}.$$

Approximation for ${}^*U_i$ or U values at full grid points are performed in an analogous manner as ${}^*h$, with the first order upwind approximation as follows

$$^*U_i = \begin{cases} U_{i-\frac{1}{2}}, & \text{if } \bar{q}_i \geq 0 \\ U_{i+\frac{1}{2}}, & \text{if } \bar{q}_i < 0. \end{cases} \tag{8}$$

LANDSLIDE DYNAMIC AND KINEMATICS

Landslide motion is based on solid block landslide theory from Watt [2]. Assuming the block is rigid and using the first Newton law's of motion, a differential equation can be derived describing the motion of the center of mass $S(t)$ parallel to the slope. The solid block inertia is determined by a sum of five external forces with components parallel to the incline: added mass force, gravitational force, buoyancy force, dynamic friction force and drag force. The differential equation can be written as

$$(M_b + \Delta M_b)\ddot{S} = (M_b - \rho_w V_b)(\sin\theta - C_n\cos\theta)g - \frac{1}{2}\rho_w C_D A_b(\dot{S})^2 \tag{9}$$

The parameter are: landslide mass M_b, added mass ΔM_b, landslide density ρ_b, water density ρ_w, landslide volume V_b, slope angle θ, gravitational acceleration g, friction coefficient C_n, hydrodynamic added mass coefficient C_d and main cross section A_c. We assume that $C_n = 0$ and equation (9) can be simplified into

$$(\gamma + C_m)\ddot{S} = (\gamma - 1)g \sin\theta - \frac{1}{2} C_D \frac{A_b}{V_b} (\dot{S})^2 \tag{10}$$

where $\gamma = \frac{\rho_b}{\rho_w}$ and $C_m = \frac{\Delta M_b}{\rho_w V_b}$.
Solution for the $S(t)$ is

$$S(t) = S_0 \ln\left(\cosh\left(\frac{t}{t_0}\right)\right). \tag{11}$$

with characteristic distance and time of motion

$$S_0 = \frac{u_t^2}{a_0} \quad \text{and} \quad t_0 = \frac{u_t}{a_0} \tag{12}$$

where a_0 is the slide initial accelerations and u_t is terminal velocity.

$$a_0 = g \sin\theta \frac{\gamma - 1}{\gamma + C_m} \tag{13}$$

and

$$u_t = \sqrt{g \sin\theta \frac{\gamma - 1}{\frac{1}{2} C_d \frac{A_b}{V_b}}} \tag{14}$$

NUMERICAL RESULT

Here we simulate wave generated by a mass sliding down as described in the previous section. The geometry of the sliding mass on the slope is shown in Figure 2. This case is originally simulated by Lynett & Liu in [4], and later by

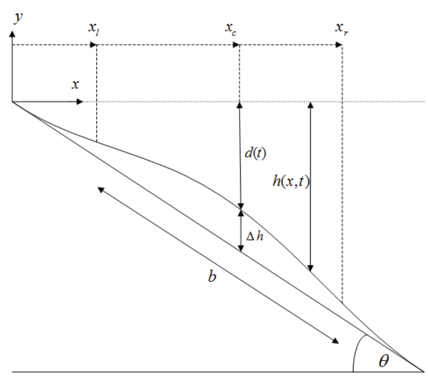

FIGURE 2. The slide mass on the slope

Fuhrman & Madsen [5]. The time history of the sliding motion, as depicted in Figure 3, is described by

$$h(x,t) = [\tan\theta]x - \frac{\triangle h}{4}[1 + \tanh(2\cos\theta(x - x_l(t)))][1 - \tanh(2\cos\theta(x - x_r(t)))], \tag{15}$$

with

$$x_l(t) = x_0 + S(t)\cos\theta - \frac{b}{2}\cos\theta, \quad x_r(t) = x_0 + S(t)\cos\theta + \frac{b}{2}\cos\theta, \tag{16}$$

where θ is the angle of the slope, $\triangle h$ is the maximum vertical height of the slide, x_l is the location of the inflection point of the left side, x_r is the location of the inflection point of the right side, and b is the length along the slope

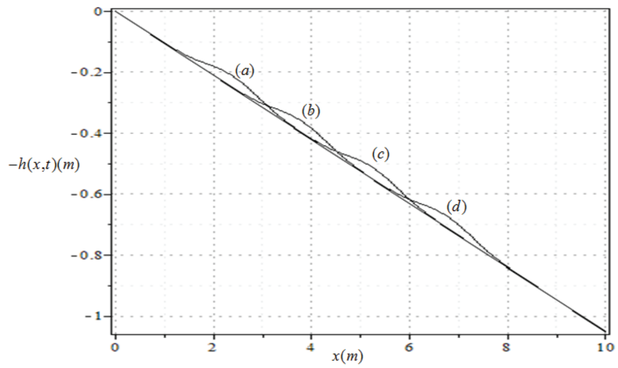

FIGURE 3. Sliding mass at (a) $t = 1.51$, (b) $t = 3.00$, (c) $t = 4.51$ and (d) $t = 5.86$.

between x_l and x_r. Here, x_0 is the initial x position of the sliding mass. We use the same parameters as in [3, 4, 5], and those are $b = 1$ m, $\triangle h = 0.05$ m, $\theta = 6^0$, $x_0 = 2.379$ m, $S_0 = 4.712$ m, and $t_0 = 3.713$ s. The scheme is implemented on a computational domain $[0, 8]$, with $\Delta x = 0.05$, $\Delta t = 0.005$.

The result at four time observations are depicted in Figure 4. It is shown that our scheme can simulate the moving shoreline correctly, and our results are comparable with the nonlinear and dispersive BIEM results of Lynett & Liu, as in [4].

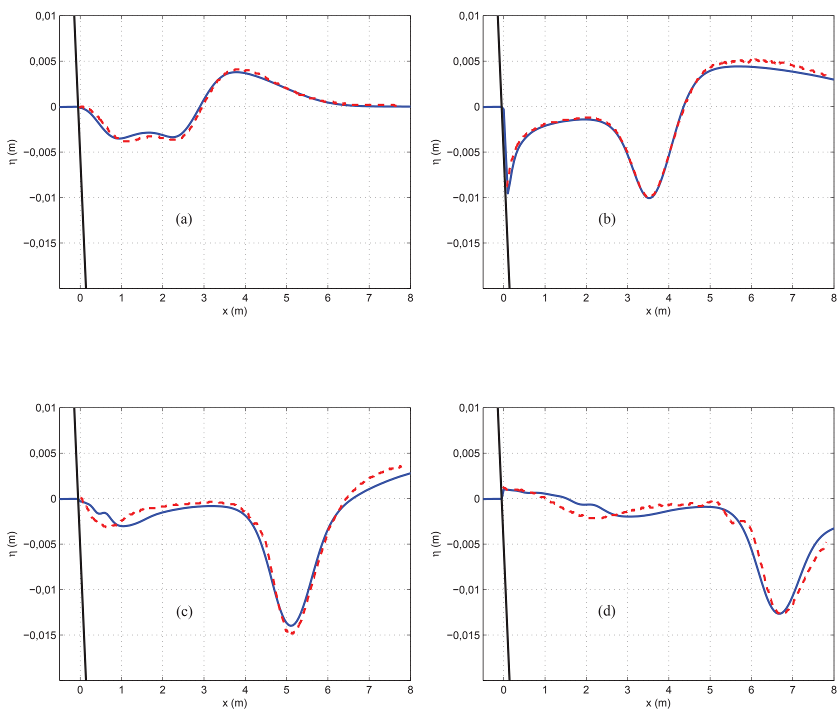

FIGURE 4. Comparison of surface elevation resulting from a landslide bottom motion between nonlinear dispersive SWE model (full lines) with nonlinear dispersive BIEM model (dash lines) at (a) $t = 1.51$, (b) $t = 3.00$, (c) $t = 4.51$, (d) $t = 5.86$.

In other simulation, we use triangle box as a rigid mass slide down an inclined plane with a 45^0 slope as shown in Figure 5. The bottom function is given by

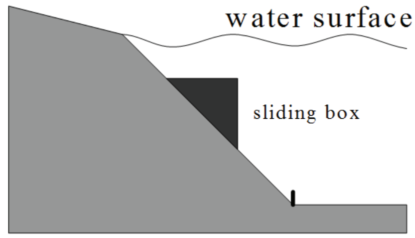

FIGURE 5. The triangle slide mass on the slope

$$h_{x,t} = \begin{cases} 1.25 - 0.25x\,, & \text{if } 0 \leq x < 1 \\ 2 - x\,, & \text{if } 1 \leq x < x_l(t) \\ 2 - x_l(t)\,, & \text{if } x_l(t) \leq x < x_r(t) \\ 2 - x\,, & \text{if } x_r(t) \leq x < 2 \\ 0\,, & \text{if } x \geq 2. \end{cases} \tag{17}$$

We use the same $S(t)$ in equation 11 as motion of the center of the triangle mass. The parameters are $b = 0.5\sqrt{2}$ m, $\theta = 45^0$, $x_0 = 0.26$ m, $S_0 = 1.4999$ m, and $t_0 = 0.8058$ s. The scheme is implemented on a computational domain $[0, 8]$, with $\Delta x = 0.01$, $\Delta t = 0.001$. The result at four time observations are depicted in Figure 6 and we compare the result with Kurganov & Petrova result. It is shown our results are good agreement with the nonlinear cental-upwind scheme results of Kurganov & Petrova , as in [7].

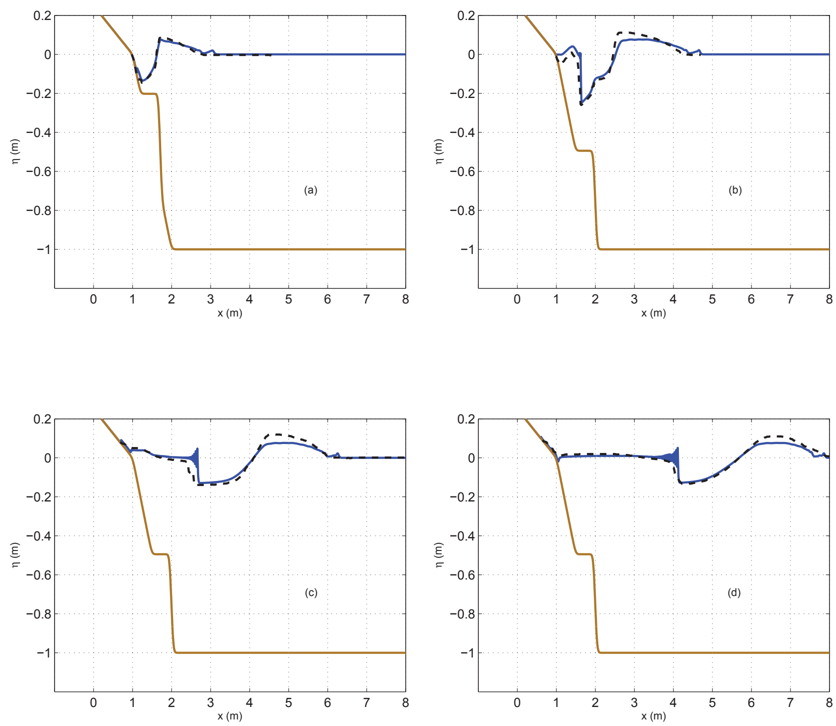

FIGURE 6. Comparison of surface elevation resulting from a triangle landslide bottom motion between nonlinear dispersive SWE model (full lines) with nonlinear central-upwind schmeme model (dash lines) at (a) $t = 0.5$, (b) $t = 1.0$, (c) $t = 1.5$, (d) $t = 2.0$.

CONCLUSIONS

The staggered conservative scheme has been successfully implemented for simulating surface wave generation by a submarine landslide. The generated free surface waves from a smooth hump show a good agreement with Lynet and Liu result from BIEM model. For landslide simulation of a triangle, our numerical result compares well against the result of Kurganov and Petrova with central-upwind scheme.

REFERENCES

[1] P. Heinrich, Nonlinear water waves generated by submarine and aerial landslide, J. Waterw. Port. Coast. Ocean Eng. 118 (1992) 249-266.

[2] P. Watts, Water waves generated by underwater landslides, Ph.D. Dissertation, California Institute of Technology, Pasadena, California, 1997

[3] S. T. Grilli and P. Watts, Modeling of waves generated by a moving submerged body. Applications to underwater landslides, Engineering Analysis with Boundary Elements 23 (1999) 645–656.

[4] P. Lynett and P. L. F. Liu, A numerical study of submarine-landslide-generated waves and run-up, Proc. R. Soc. Lond. A 458 (2002) 2885–2910.

[5] D. R. Fuhrman, P. A. Madsen, Tsunami generation, propagation, and run up with high order Boussinesq model, Coastal Engineering 56 (2009) 747-758.

[6] G. S. Stelling, S. P. A. Duinmeijer, A staggered conservative scheme for every Froude number in rapidly varied shallow water flows, International Journal For Numerical Methods in Fluids 43 (2003) 1329-1354.

[7] A. Kurganov, G. Petrova, A central-upwind scheme for nonlinear water waves generated by submarine landslide, in: Hyperbolic Problems: Theory, Numerics, Application (2008) 635-642.